(a) You’re working with two random variables $X$ and $Y$, which may be dependent and for which $V(X) = V(Y)$. Show that the random variables $W = (X+Y)$ and $Z = (X-Y)$ are uncorrelated. *Hint:* Nothing fancy — just simplify the covariance of $W$ and $Z$, using properties of covariance we’ve talked about in class.

(b) You’re working with two random variables $X$ and $Y$ that are negatively correlated. Which is bigger — $V(X+Y)$ or $V(X-Y)$ — or are they equal? Show your calculations.

(c) You’re working with two random variables $X$ and $Y$ such that $V(X) = 9$, $V(Y) = 4$, and $\rho(X,Y) = -\frac{1}{6}$. Compute $V(X+Y)$ and $V(X-Y)$ (show your calculations).

(d) You and your research assistant (RA) are working with two random variables $X$ and $Y$, and your RA has computed the following values: $E(X) = 3$, $E(Y) = 2$, $E(X^2) = 10$, $E(Y^2) = 29$, and $E(XY) = 0$. Show that there must be something wrong in this computation. *Hint:* Consider the bounds on variances and correlations.