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AMS 131: Quiz 5

Name: _____ Discussion Section (Day and Time): _____

(You can use the back of this page as needed.)

You're working on a problem involving two continuous random variables X and Y, and you figure out that their joint PDF has the following form:

$$f_{X,Y}(x,y) = \left\{ \begin{array}{cc} \frac{15}{4}x^2 & \text{for } 0 \le y \le 1 - x^2 \\ 0 & \text{otherwise} \end{array} \right\} . \tag{1}$$

(a) Sketch the support S of this bivariate distribution.

(b) What double integral would you have to compute to verify that this joint PDF does indeed integrate to 1? Explain briefly.

(c) It can be shown that the marginal PDFs of X and Y with this joint PDF are

$$f_X(x) = \left\{ \begin{array}{cc} \frac{15}{4}x^2(1-x^2) & \text{for } -1 \le x \le 1 \\ 0 & \text{otherwise} \end{array} \right\} \tag{2}$$

and

$$f_Y(y) = \left\{ \begin{array}{cc} \frac{5}{2}(1-y)^{\frac{3}{2}} & \text{for } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{array} \right\} . \tag{3}$$

Pick one of these two marginals and verify that it's correct.

(d) Are X and Y independent in this joint distribution? Explain briefly.