6. (public health) The Enzyme-Linked ImmunoSorbent Assay (ELISA) test was approved by many countries around the world in the mid-1980s to screen donations to blood banks for the presence of the Human Immunodeficiency Virus (HIV), which causes AIDS. ELISA works by detecting antibodies (substances that the body produces when the virus is present), but (as with any screening test) in practice it makes some mistakes. Many versions of the ELISA test are now commercially available, and — in addition to their use in blood banks — these tests are now also sometimes used to screen for HIV in the general population (as we'll see, using only a single ELISA test for this purpose without any other testing may not be such a good idea). In 2004 the World Health Organization surveyed¹ the accuracy of 10 of the simplest and quickest of the ELISA tests on the market. In this problem we'll look at the performance of the Efoora HIV Rapid test, which for simplicity will simply be referred to here as ELISA.

ELISA was designed so that when a given blood sample does in fact contain HIV, the test gives a positive result (that is, ELISA reports that in its opinion this blood sample has HIV in it) 96% of the time: this is referred to as the *sensitivity* of a screening test. Moreover, when the blood being tested does not have the virus ELISA will announce a negative result 97% of the time: this is ELISA's *specificity*. According to the Centers for Disease Control and Prevention, the *prevalence* of HIV in the population of Americans who are 18 years old or older² (the proportion of these people who have HIV) is currently thought to be about 0.4%.

- (a) Let $A = \{\text{person has HIV}\}, + = \{\text{ELISA positive}\}, \text{ and } = \{\text{ELISA negative}\}.$
 - (i) Thinking about the process of choosing one person at random from the population of Americans who are 18 years old or older, express the three numerical facts above (sensitivity, specificity and prevalence) in unconditional and conditional probability terms. [6 points]
 - (ii) Use those same three numerical facts, starting with the 0.4% prevalence, to fill in the 2 by 2 contingency table below on a hypothetical set of 100,000 blood samples, briefly explaining your reasoning (on the back of this page). [8 points]

		The Truth	
			Person
		Person	Doesn't
		Has HIV	Have HIV
		(A)	$(\mathrm{not}\ A)$
ELISA Diagnosis	ELISA positive (+)		
	ELISA negative (-)		

100,000

¹See the WHO web site for details.

²See hivinsite.ucsf.edu/InSite?page=kb-01-03 for details.

(b) Use the completed table to show that if someone donates blood and the ELISA test comes out negative, the probability the person does not in fact have HIV given this negative result is virtually 100%, but if ELISA comes out positive the probability the person actually has HIV is only about 11%! [6 points]

(c) Explain these results by identifying the two kinds of mistakes ELISA could make and discussing their implications from the blood bank's point of view (when the test is used, for example, for screening donated blood at a hospital). [6 points]

(d) In practice it's possible to "tune" screening tests like ELISA by changing the threshold of antibodies required to announce a positive result, which will act on the 96% sensitivity and 97% specificity values mentioned above in a tug-of-war fashion: you can increase the sensitivity, for instance, but only by allowing the specificity to decrease (and vice versa). If people insist on using ELISA to screen for HIV in the general population (ignoring for the moment the use of this test by blood banks), which would be better to increase: ELISA's sensitivity or specificity? Explain briefly. (There is no single clear-cut answer here; a good argument either way will get full credit.) [4 points]

Note: In all four parts of this problem it's not necessary for your answers to completely fill up all of the space given on the exam paper — good answers to all four parts only require a few sentences each.