This a case study; time: experiments; next events: sample time: space, set theory.

Tu Th 5:35 - 9:45 am
Baskin Auditorium
Tu Th 2:15 - 3:25 pm
Baskin Auditorium

video for AMS 171

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Re: Thomas Bayes
(1750) Bayesian

Laplace - 1775 - 1825
Gauss - 1850

Venn (1850) frequentist (relative frequency)

Pascal, Fermat (classical) 1650

AMS 131
31 Jul 2017
population

\[ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

equally likely

\[ g \rightarrow Y_1 \]

sample

\[ \frac{1}{3} \]

\( \Pr ( Y_1 = 2 ) = \frac{1}{3} \)

\( \{ \text{unsampled} \} \)

\( \{ \text{sampled} \} \)

elements

\( \{ \text{outcomes} \} \)

\( (E_{05}) \)

want representative:

\( \{ \text{sampled} \} \) & \( \{ \text{unsampled} \} \)

\( p.p. \) elements should be as similar as possible in all relevant ways

equally likely model (ELM)

\[ \Pr (1 \text{ or more } T-5) = ? \]

\[ \Pr (A \text{ or } B) = \Pr (A) \times \Pr (B) \]

\[ \Pr (\text{not } A) = \? \]

\[ \Pr (A) \]
\[ P(A \text{ and } B) = P(A) \times P(B) \]

\[ P(\text{not } A) = 1 - P(A) \]

\[ P(A) + P(\text{not } A) = 1 \]

\[ 0 \leq P(A) \leq 1 \]

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
If \( P(A \text{ and } B) = 0 \), people say \( A, B \) are mutually exclusive.

Special case:

If \( A, B \) mutually exclusive,

\[
P(A \text{ or } B) = P(A) + P(B)
\]

\[\begin{bmatrix}
1 \\
2 \\
9
\end{bmatrix}
\]

Sample

\[
\begin{bmatrix}
\gamma_1 \\
\gamma_2
\end{bmatrix}
\]

\( n = 2 \)

\[P(\gamma_1 = 9 \text{ and } \gamma_2 = 9) = ?\]

At random with replacement = independent identically distributed (IID)

At random without replacement
at random without replacement = simple random sampling (SRS)

\[ \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \]

\[ \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \]

\[ \text{IID} \]

\[ \text{EEM} \]

\[ p(\gamma_1 = 9 \text{ and } \gamma_2 = 9) = \frac{1}{9} \]

\[ p(\gamma_1 = 9) \cdot p(\gamma_2 = 9) = \frac{1}{3} \cdot \frac{1}{3} = \frac{3}{9} \cdot \frac{3}{9} \]

conjecture:

\[ p(A \text{ and } B) = p(A) \cdot p(B) \]

works for IID, fails for SRS
$$\Pr(\gamma_1 = 9 \text{ and } \gamma_2 = 9) = 0$$

$$\Pr_{\text{SRS}}(\gamma_1 = 9) = \frac{1}{3} = \frac{2}{6}$$

$$\Pr_{\text{SRS}}(\gamma_2 = 9) = \frac{2}{6} = \frac{1}{3}$$

$$\Pr(\gamma_1 = 9 \text{ and } \gamma_2 = 9) \neq \Pr_{\text{SRS}}(\gamma_1 = 9) \cdot \Pr_{\text{SRS}}(\gamma_2 = 9)$$

$$= \frac{1}{3} \cdot \frac{1}{3}$$

**Bayes' Rule:**

$$\Pr(B|A) = \frac{\Pr(A \text{ and } B)}{\Pr(A)}$$

Both sides by $\Pr(A)$:

$$\Pr(A \text{ and } B) = \Pr(A) \cdot \Pr(B|A)$$
\[ P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \]

\[ P(A \text{ and } B) = P(B) \cdot P(A \mid B) = P(A) \cdot P(B \mid A) \]

"General product rule" (for and)

\[ P_{\text{sr}}(Y_1 = 9 \text{ and } Y_2 = 9) = \]

\[ P_{\text{sr}}(Y_1 = 9) \cdot P_{\text{sr}}(Y_2 = 9 \mid Y_1 = 9) = \frac{1}{3} \cdot 0 = 0 \]

\[ \text{Frequentist definition of independence: } A, B \text{ independent} \]

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

\[ \text{IID} \Rightarrow \text{independent} \]
with

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

\[ = P(A) \cdot P(B | A) \]

\[ = P(B) \cdot P(A | B) \]

so under

IID sampling \[ P(B) = P(B | A) \]

and \[ P(A) = P(A | B) \]

under IID sampling, into about 1st draw doesn't change chances of second draw, & vice versa

Bayesian definition of independence: \( A, B \) indep.

if into about 1 doesn't change chances of other
\[ P(1 \text{ or more TS babies}) = 1 - P(\text{no TS babies}) \]
\[ = 1 - P(\text{not TS on 1st} \cap \text{not TS on 2nd} \cap \ldots \cap \text{not TS on 5th}) \]
\[ = 1 - P(\text{not TS on 1st}) \cdot P(\text{not TS on 2nd}) \cdots P(\text{not TS on 5th}) \]
\[ = 1 - \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right) \]
\[ = 1 - \left(1 - \frac{1}{4}\right)^5 = 76\% \]

\[ n = 4 \text{ children} \]
\[ p = \text{prob of bad thing happening} \]
\[ P(1 \text{ or more bad things occurring}) = 1 - (1-p)^n \]
alternative argument: 6 EoE, equipossible, so ELM

\[ P(A) = \frac{5}{6} \]

\[ = 83\% \]

for more. Because 6 possibilities are not equally likely