AMS 131

31 Jul 2017

read: DeGroot & Schervish (DS) ch. 1

this time: a case study; experiments

next time: events, sample space, set theory

TuTh 8.35-9.45am Baskin Auditorium

TuTh 11.30am - 12.40pm Baskin 165

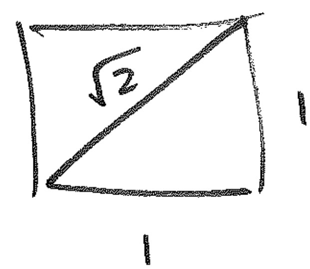
TuTh 2.15 - 3.25pm Baskin Auditorium

webcast.ucsc.edu

video for AMS 131

user: ams-131

password: asdfghjk



Rev. Thomas Bayes (~1750) Bayesian

Pascal, Fermat (classical) 1650

Laplace, Gauss $\}$ ~1775-1825

Venn (1850)

frequentist (relative frequency)

population

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

equipossible

at random

sample

$$\begin{bmatrix} Y_1 \end{bmatrix} \quad n = 1$$

ELM ✓

$$P(Y_1 = 2) = \frac{1}{3}$$

most representative:

(S) sampled & (U) unsampled pop. elements should be as similar as possible in all relevant ways

pop

U unsample | S sample

elementary outcomes (EOs)

equally likely model (ELM)

we want $P(1 \text{ or more T-5}) = ?$

$$P(A \text{ or } B) \overset{?}{=} P(A) \ ? \ P(B)$$

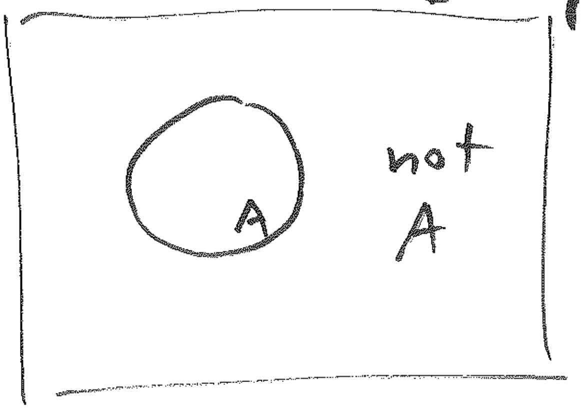
$$P(\text{not } A) \overset{?}{=} P(A)$$

$P(A \text{ and } B) \overset{?}{=} P(A) \overset{?}{\cdot} P(B)$

not

Venn diagram

all possibilities



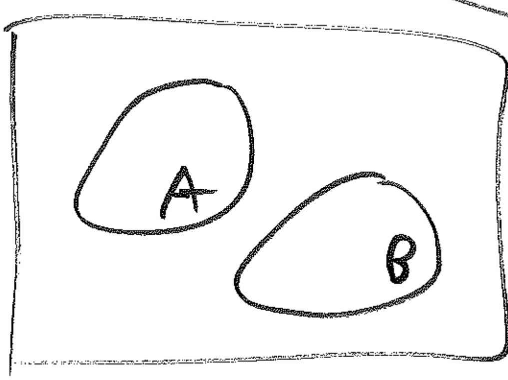
$P(\square) = 1$

$P(A) + P(\text{not } A) = 1$

$0 \le P(A) \le 1$

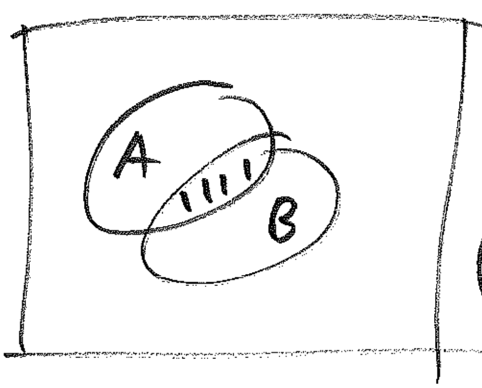
$$\boxed{\underbrace{P(A)}_{\text{direct}} = \underbrace{1 - P(\text{not } A)}_{\text{indirect}}}$$

or



$P(A \text{ or } B) =$

$P(A) + P(B)$



general addition rule for or

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

if $P(A \text{ and } B) = 0$, people say A, B are <u>mutually exclusive</u> (no overlap)

special case:

if A, B mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B)$$

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and

$$\begin{array}{ccc} \text{pop} & & \text{sample} \\ \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} & \xrightarrow{\text{at random}} & \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad n = 2 \end{array}$$

$$P(y_1 = 9 \text{ and } y_2 = 9) = \,?$$

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at random with replacement = independent identically distributed (IID)

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at random without replacement

at random without replacement = simple random sampling (SRS)

ELM ✓

IID

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{IID}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) \underset{\text{IID}}{=} ?$$

(EOS)

1st draw \ 2nd draw	1	2	9
1	(1, 1)	(1, 2)	(1, 9)
2	(2, 1)	(2, 2)	(2, 9)
9	(9, 1)	(9, 2)	(9, 9)

$$= \frac{1}{9} =$$

$$P(Y_1 = 9) \cdot P(Y_2 = 9)$$

$$\frac{1}{3} \cdot \frac{1}{3}$$

$$= \frac{3}{9} \cdot \frac{3}{9}$$

conjecture:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

works for IID, fails for SRS

SRS

1st \ 2nd	1	2	9
1	~~(1,1)~~	(1,2)	(1,9)
2	(2,1)	~~(2,2)~~	(2,9)
9	(9,1)	(9,2)	~~(9,9)~~

ELM ✓

$$P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) = 0$$

$$P_{SRS}(Y_1 = 9) = \frac{1}{3} = \frac{2}{6}$$

$$P_{SRS}(Y_2 = 9) = \frac{2}{6} = \frac{1}{3}$$

$$P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) = 0$$

$$\neq P_{SRS}(Y_1 = 9) \cdot P_{SRS}(Y_2 = 9)$$

$$= \frac{1}{3} \cdot \frac{1}{3}$$

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multiply $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$

both sides by $P(A)$:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

conditional probability

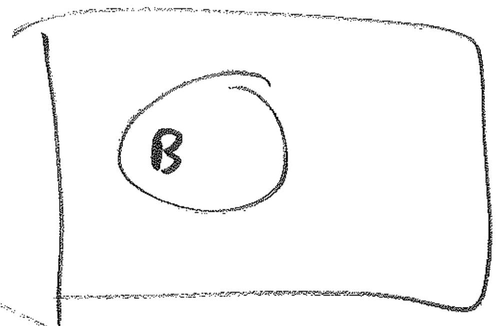
Abraham de Moivre (~1725)

Thomas Bayes (~1750)

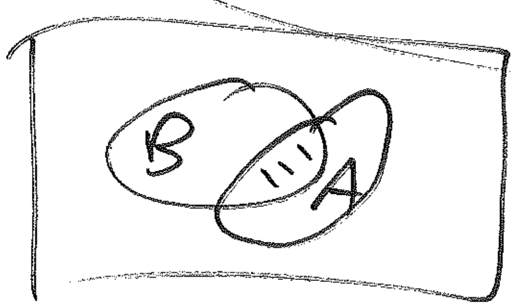
def: $P(\,B \text{ given } A\,) = ?$

$P(\,B \mid A\,) = ?$

$$P(B) = \frac{B}{\square} = \frac{}{1}$$



$$P(B \mid A) = \frac{(=)\ A \text{ and } B}{A}$$



def.

$$P(B \mid A) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & \text{if } P(A) = 0 \end{cases}$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{so}$$

$$\boxed{\begin{aligned} P(A \text{ and } B) &= P(B) \cdot P(A \mid B) \\ &= P(A) \cdot P(B \mid A) \end{aligned}}$$

general product rule for $\boxed{\text{and}}$

"chain rule"

$$P_{SRS}(\overbrace{Y_1 = 9}^{A} \text{ and } \overbrace{Y_2 = 9}^{B}) =$$

$$P_{SRS}(\overset{A}{Y_1 = 9}) \cdot P_{SRS}(\overset{B}{Y_2 = 9} \overset{\mid}{\mid} \overset{A}{Y_1 = 9})$$

$$= \frac{1}{3} \cdot 0 = 0 \checkmark$$

~~frequentist definition of~~ independence: A, B indep. if

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B)}$$

with IID = independent

with IID $P(A \text{ and } B) = P(A) \cdot P(B)$

$$= P(A) \cdot P(B \mid A)$$

$$= P(B) \cdot P(A \mid B)$$

so under IID sampling $P(B) = P(B \mid A)$

and $P(A) = P(A \mid B)$

under IID sampling, info about 1st draw doesn't change chances of second draw, & vice versa

Bayesian definition of independence: A, B indep. if info about 1 doesn't change chances of other

$\boxed{TS}$ A

$p(\text{1 or more TS babies})$

$= 1 - p(\text{no TS babies})$

$= 1 - p\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on 1st}\end{matrix} \text{ and } \begin{matrix}\text{not}\\ \text{TS}\\ \text{on 2nd}\end{matrix} \text{ and } \cdots \text{ and } \begin{matrix}\text{not}\\ \text{TS}\\ \text{on 5th}\end{matrix}\right)$

$\overset{\text{indep. (I)ID}}{=} 1 - p\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on 1st}\end{matrix}\right) \cdot p\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on 2nd}\end{matrix}\right) \cdots p\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on 5th}\end{matrix}\right)$

$\overset{\text{iden. dist. I(ID)}}{=} 1 - \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right)$

$= 1 - \left(1 - \frac{1}{4}\right)^5 \doteq 76\%$

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n = # children

p = prob of bad thing happening

$p(\text{1 or more bad things (IID)}) = 1 - (1-p)^n$

TS kids
0
1
2
3
4
5

A: 1 or more

alternative argument: 6 EOs, equipossible, so ELM

applies: $P(A) = \frac{5}{6}$

$\doteq 83\%$

bogus because 6 possibilities are not equally likely