A case study in permutation and combinations.

\[ P(\text{infected in } n \text{ unprotected acts}) \]

\[ p = P(\text{infection on any single act}) = \frac{1}{500} \]

\[ \star = 1 - p(\text{not inf. in n acts}) \]

\[ = 1 - P(\text{not inf. on 1st} \land \text{not inf. on 2nd} \land \cdots \land \text{not inf. on } n) \]

\[ \text{(ID)} \]

\[ = 1 - P(\text{not inf. on 1st}) \cdot P(\text{not inf. on 2nd}) \cdots P(\text{not inf. on } n) \]

\[ \text{(ID)} \]

\[ = 1 - \left( 1 - p \right) \cdot \left( 1 - p \right) \cdots \left( 1 - p \right) \]

\[ = 1 - (1 - p)^n \]
<table>
<thead>
<tr>
<th>$n$</th>
<th>Dr. Schram</th>
<th>correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p = \frac{1}{500}$</td>
<td>$1$</td>
</tr>
<tr>
<td>100</td>
<td>0.20 = 20%</td>
<td>$1 - (1 - \frac{1}{500})^{100} = 0.18$</td>
</tr>
<tr>
<td>500</td>
<td>1 = 100%</td>
<td>$1 - (1 - \frac{1}{500})^{500} = 0.63$</td>
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</table>

$$n \cdot np = 1 - (1 - p)^n$$

$$p_i = P(\text{inf from partner } i)$$

$$P(\text{inf in n sets with n partners}) = 1 - (1 - p_1)(1 - p_2) \cdots (1 - p_n)$$

$$= 1 - \prod_{i=1}^{n} (1 - p_i)$$
Dr. John Doll (epidemiologist) predicted for each country

% of smokers

(1957)

% dying of heart disease

positive association

1925

F = Fisher's hypothesis

F

\[ P(\text{HHT} | F) = P(\text{H on 1st} \cap \text{H on 2nd} | F) \]

\[ = P(\text{H on 1st}) \cdot P(\text{H on 2nd}) \]

\[ = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \]

in conclusion

because 25% is still fairly large

\[ \approx 25\% \]
\[ P(\text{HHHHHHHHH} \mid F) = \left( \frac{1}{2} \right)^9 \]

\[ = \frac{1}{512} = 0.2\% \]

if \( F \) true, data very unlikely;

therefore \( F \) probably false.

\textit{Probabilistic proof by contradiction}

much used in statistics (9.43)

\[ \mathcal{S} \text{ if } B_1, \ldots, B_n \text{ are mutually exclusive and exhaustive} \]

\[ (\bigcup_{i=1}^{n} B_i = \mathcal{S}), \{B_1, \ldots, B_n\} \]

\textit{is said to have formed a partition of } \mathcal{S}. \]
\[ A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n) \]

\[ = (A \text{ and } B_1) \lor (A \text{ and } B_2) \lor \ldots \lor (A \text{ and } B_n) \]

\[ P(A) = P\left[ (A \text{ and } B_1) \lor (A \text{ and } B_2) \lor \ldots \lor (A \text{ and } B_n) \right] \]

\[ = \sum_{i=1}^{n} P(A \text{ and } B_i) \]

\[ P(A \text{ and } B_i) = P(B_i) \cdot P(A \mid B_i) \]

So

\[ P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A \mid B_i) \]

Law of Total Probability (LTP)

\[ \text{(LTP)} \]
if $A_1 = A_2$ then

$p(A_1) = p(A_2)$

Continuity of $p(\cdot)$

Desirable interesting fact:

$p_k \left( \bigcup_{i=1}^{\infty} A_i \right) = \sum_{i=1}^{\infty} p_k(A_{i})$

$\leftrightarrow$ continuity

(10.42)