this time: case studies; permutations
next time: & combinations; axioms

read DS ch. 1

AMS131

2 Aug 17

Dr. Schram case study ①

$P(\text{infected in } n \text{ unprotected acts}) = ?$ (*)

$p = P(\text{infection on any single act}) = \frac{1}{500}$

$(*) = 1 - P(\text{not inf. in } n \text{ acts})$

$= 1 - P\left(\begin{matrix}\text{not}\\ \text{inf.}\\ \text{on 1st}\end{matrix} \text{ and } \begin{matrix}\text{not}\\ \text{inf.}\\ \text{on 2nd}\end{matrix} \text{ and } \cdots \text{ and } \begin{matrix}\text{not}\\ \text{inf.}\\ \text{on } n\text{th}\end{matrix}\right)$

$\overset{(\mathrm{I})\,\mathrm{ID}}{=} 1 - \underline{P\left(\begin{matrix}\text{not}\\ \text{inf.}\\ \text{on 1st}\end{matrix}\right)} \cdot \underline{P\left(\begin{matrix}\text{not}\\ \text{inf.}\\ \text{on}\\ \text{2nd}\end{matrix}\right)} \cdot \ldots \cdot \underline{P\left(\begin{matrix}\text{not}\\ \text{inf.}\\ \text{on } n\text{th}\end{matrix}\right)}$

$\overset{\mathrm{I}(\mathrm{ID})}{=} 1 - (1-p) \cdot (1-p) \cdot \ldots \cdot (1-p)$

$= 1 - (1-p)^n$

n	Dr. Schram	correct
1	$p = \frac{1}{500}$	p
100	$0.20 = 20\%$	$1-\left(1-\frac{1}{500}\right)^{100} = 0.18$
500	$1 = 100\%$	$1-\left(1-\frac{1}{500}\right)^{500} \doteq 0.63$
n	np	$1-(1-p)^n$

$$= 1-(1-np+\cdots)$$

$$= np - \boxed{}$$

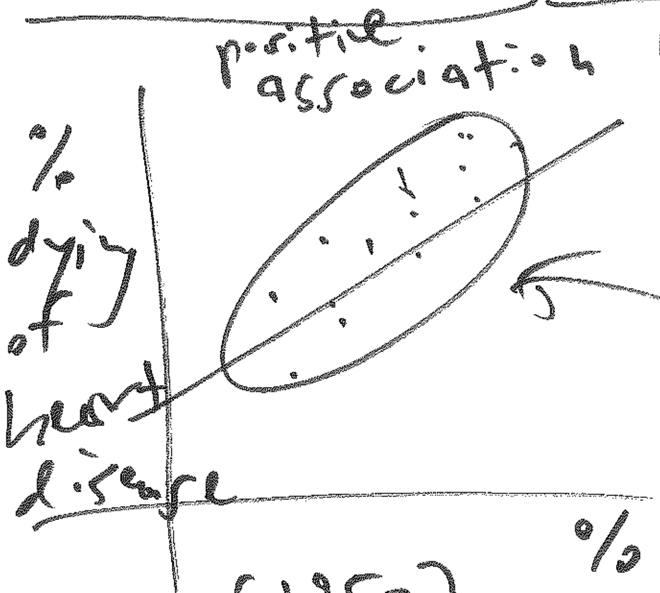
$p_i = P(\text{you inf. from partner } i)$

P(inf in n acts with n partners)

$$= 1-(1-p_1)(1-p_2)\cdots(1-p_n)$$

$$= 1-\prod_{i=1}^{n}(1-p_i)$$

prof. Sir
R.A. Fisher FRS (1890 – 1962) $\boxed{\sim 1925}$ ③



(1957)

Dr. John Doll
(epidemiologist)

1 point for each country

F = Fisher's hypothesis

H = heads
T = tails

$$P(HH \mid F) = P(H \text{ on } 1^{st} \text{ and } H \text{ on } 2^{nd} \mid F)$$

lung cancer

$$= P(H \text{ on } 1^{st}) \cdot P(H \text{ on } 2^{nd})$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

inconclusive

$$= 25\%$$

because 25% is still fairly large

$P(H\,H\,H\,H\,H\,H\,H\,H\,H \mid F) = \left(\frac{1}{2}\right)^9$

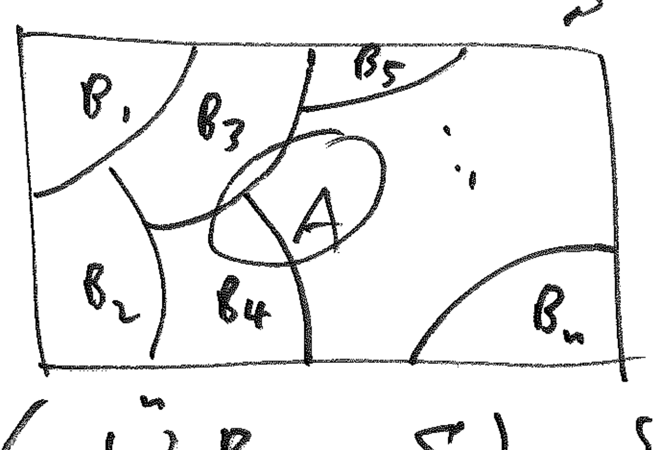
$= \frac{1}{512} \doteq 0.2\%$

if F true, data very unlikely;

therefore F probably false

probabilistic proof by contradiction

much used in statistics (9.43)



if $B_1, \ldots, B_n$ are mutually exclusive & exhaustive

$\left(\bigcup_{i=1}^{n} B_i = S\right)$, $\{B_1, \ldots, B_n\}$ is said to have formed a partition of S.

$$A = (A \cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_n)$$

$$= (A \text{ and } B_1) \text{ or } (A \text{ and } B_2) \text{ or } \dots \text{ or } (A \text{ and } B_n)$$

$$P(A) = P\left[(A \text{ and } B_1) \text{ or } (A \text{ and } B_2) \text{ or } \dots \text{ or } (A \text{ and } B_n)\right]$$

$$= \sum_{i=1}^{n} P(A \text{ and } B_i)$$

but

$$P(A \text{ and } B_i) = P(B_i) \cdot P(A \mid B_i)$$

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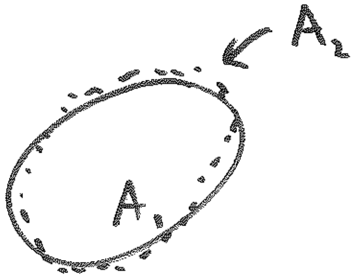
so $$\boxed{P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A \mid B_i)}$$

Law of Total Probability (LTP)



if $A_1 \doteq A_2$ then

$$p(A_1) \doteq p(A_2)$$

desirable $\boxed{\text{continuity of } p_K(\cdot)}$

interesting fact:

$$p_K\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} p_K(A_i)$$

$\leftrightarrow$ continuity (10.42)

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