Disc. Sec. 10

Case study: roulette

AMS 131 31 Aug 17

pop.: possible net gains — you bet on (6) — sample: Realized spins — repeated sampling: possible sums $ (1)

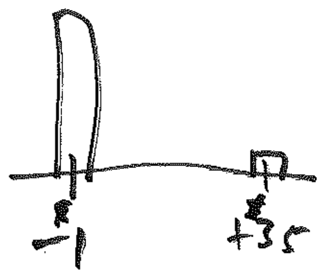
your # | net gain from single play

your #	net gain
1	−1
⋮	⋮
5	−1
6	+35
7	−1
⋮	⋮
36	−1
0	−1
00	−1

$N = 38$

mean $\mu = \$ -0.05$

SD $\sigma = \$5.76$



PMF of X_1

IID

your net gain

$$\begin{array}{c} X_1 \\ \vdots \\ X_n \end{array} \begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} \quad n = 1000$$

sum $S_n = ?$ (ex. −88)

IID

$$\begin{pmatrix} \vdots \end{pmatrix} \quad n = 1000$$

sum $S_n = ?$ (ex. +\$33)

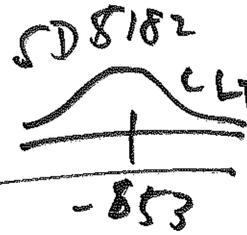
⋮

$$\begin{pmatrix} -88 \\ +33 \\ \vdots \\ \vdots \end{pmatrix} \quad m$$

long run mean $E(S_n) \doteq -\$53$

long run SD $SD(S_n) = \sqrt{V(S_n)} \doteq 8182$

long run PDF



$$\text{skewness}(\bar{X}_n) = \text{skewness}(S_n)$$

$$= \frac{\text{skewness}(X_1)}{\sqrt{n}}$$

$$\mu = \frac{37(\$-1) + 1(\$+35)}{38} = \frac{\$-2}{38} \doteq \$-0.05$$

$$\sigma = \sqrt{\frac{37\left[(\$-1) - (-0.05)\right]^2 + 1\left[(\$+35) - (-\$0.05)\right]^2}{38}}$$

$$= \$5.76$$

$$S = \sum_{i=1}^{n} X_i$$

$$E(S) = E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i) = n\mu$$

$$= (1000)(\$-0.0526) \doteq -\$53$$

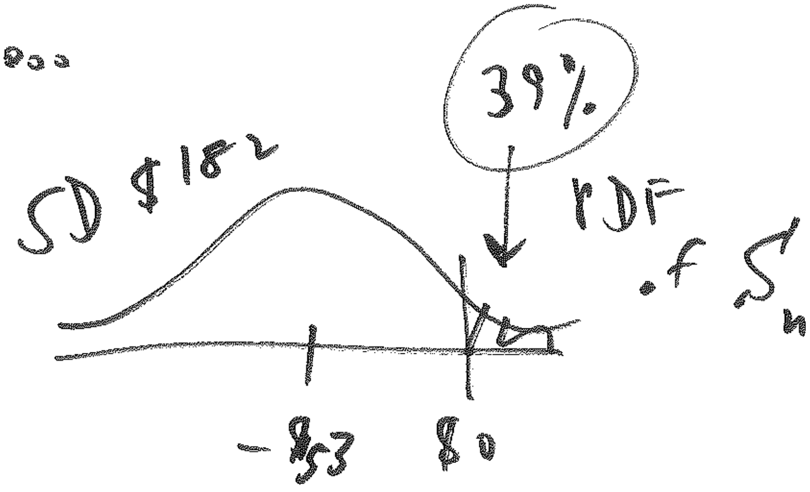
$$V(S) = V\left(\sum_{i=1}^{n} X_i\right) \overset{(I)}{=} \sum_{i=1}^{n} V(X_i) \overset{I(D)}{=} \sum_{i=1}^{n} \sigma^2 = n\sigma^2 \;*$$

$$SD(S) = \sigma\sqrt{n} = \$5.76\sqrt{1000}$$

$$= \$182$$

$\$-53 \pm 2 \cdot \$182$

possible values for S



$$\frac{\$0 - (-\$53)}{\$182} \doteq +0.29$$

P(coming out ahead with n $1 bets)

$= P(S_n > 0)$

$= ?$

X_1 = your net gain on spin 1

$$f_{X_1}(x_1) = \begin{cases} \frac{37}{38} & \text{for } x_1 = -1 \\ \frac{1}{38} & x_1 = +35 \\ 0 & \text{else} \end{cases}$$

$$\text{kurtosis}(\bar{X}_n) = \text{kurtosis}(S_n)$$

$$= \frac{\text{kurtosis}(X_1)}{n}$$

$$P(S_n > 0) = ?$$

$$S_n = \sum_{i=1}^{n} X_i$$

$$X_i = \begin{cases} -1 & \text{with prob. } \frac{37}{38} \\ +35 & \frac{1}{38} \end{cases}$$

$$\frac{X_i + 1}{36} = \begin{cases} 0 & \text{w.p. } \frac{37}{38} \\ 1 & \frac{1}{38} \end{cases}$$

$$Y_i = \frac{X_i + 1}{36} \overset{IID}{\sim} \text{Bernoulli}\left(\frac{1}{38}\right)$$

$$T_n = \sum_{i=1}^{n} I_i \sim \text{Binomial}\left(n, \frac{1}{38}\right)$$ (5)

$$T_n = \sum_{i=1}^{n} \frac{X_i + 1}{36} = \frac{1}{36} \underbrace{\sum_{i=1}^{n} X_i}_{S_n} + \frac{n}{36}$$

$$P(S_n > 0) = P\left(\frac{1}{36} S_n > 0\right)$$

$$= P\Big(\underbrace{\frac{1}{36} S_n + \frac{n}{36}}_{T_n} > \frac{n}{36}\Big)$$

so

$$P(S_n > 0) = P\Big(\underset{\substack{\uparrow \\ \text{Binomial}\left(n, \frac{1}{38}\right)}}{T_n} > \frac{n}{36}\Big)$$

$$P(\underbrace{LLL \ldots L}_{\longleftarrow 36 \longrightarrow}) = P(S_n = -\cancel{\$}36)$$

$$= \left(\frac{37}{38}\right)^{36}$$