Disc. Sec. 6

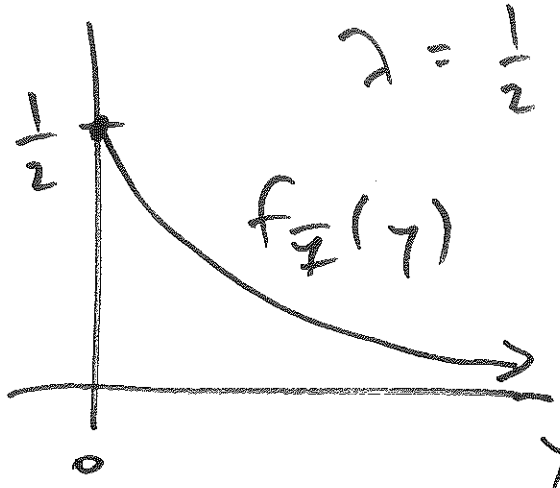
$(Y \mid \lambda) \sim \text{Exponential}(\lambda)$

AMS 131, 17 Aug 19

$\leftrightarrow$ PDF $\rightarrow$ $f_Y(y) = \begin{cases} \lambda e^{-\lambda y} & \text{for } y \geq 0 \\ 0 & \text{else} \end{cases}$ (1)

CDF

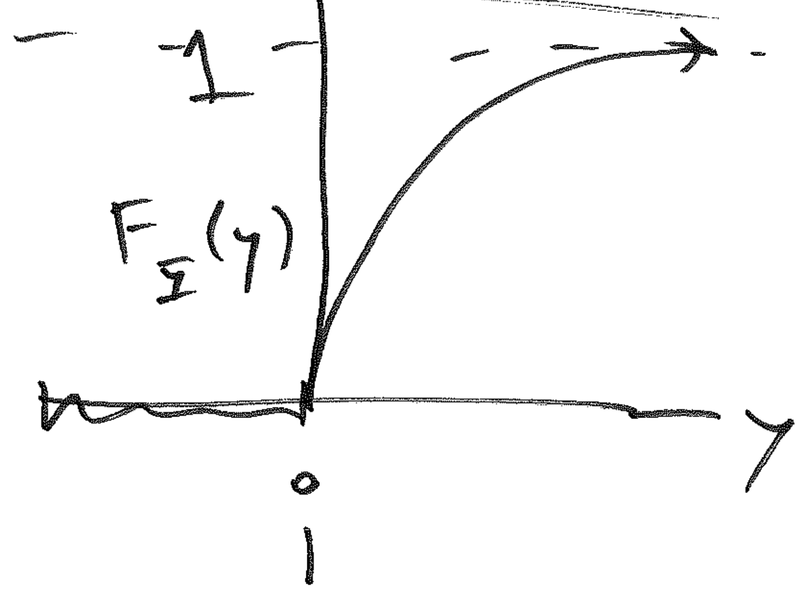
$\lambda = \frac{1}{2}$

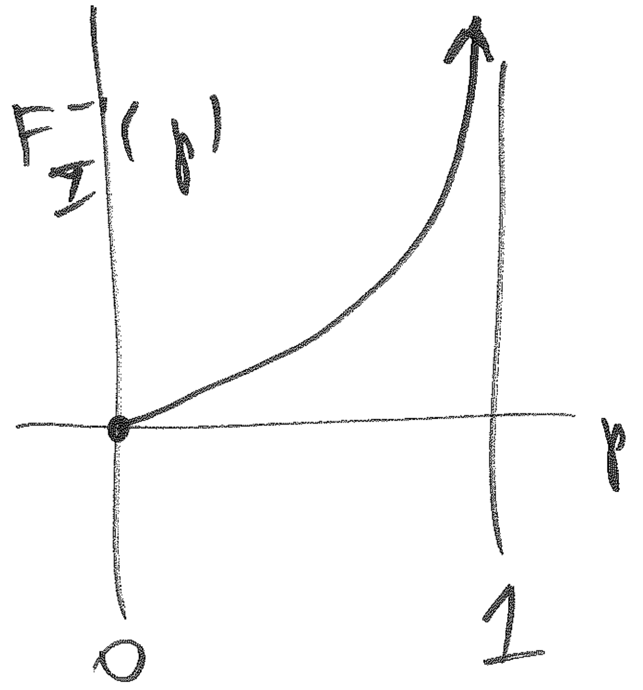


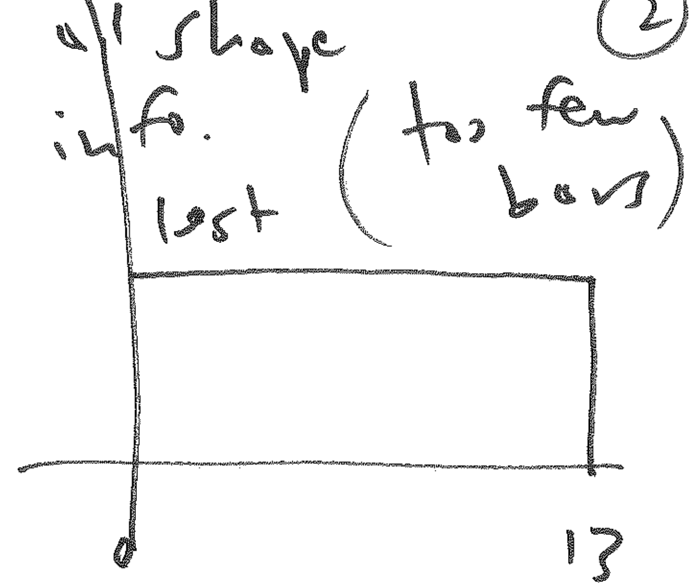
$F_Y(y) = \begin{cases} 0 & \text{for } y \leq 0 \\ 1 - e^{-\lambda y} & y \geq 0 \end{cases}$

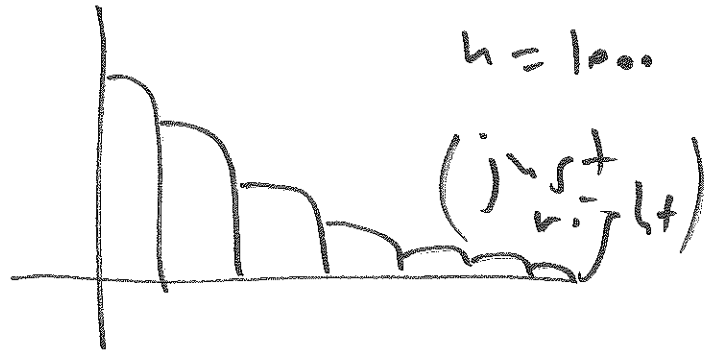
for $0 < p < 1$

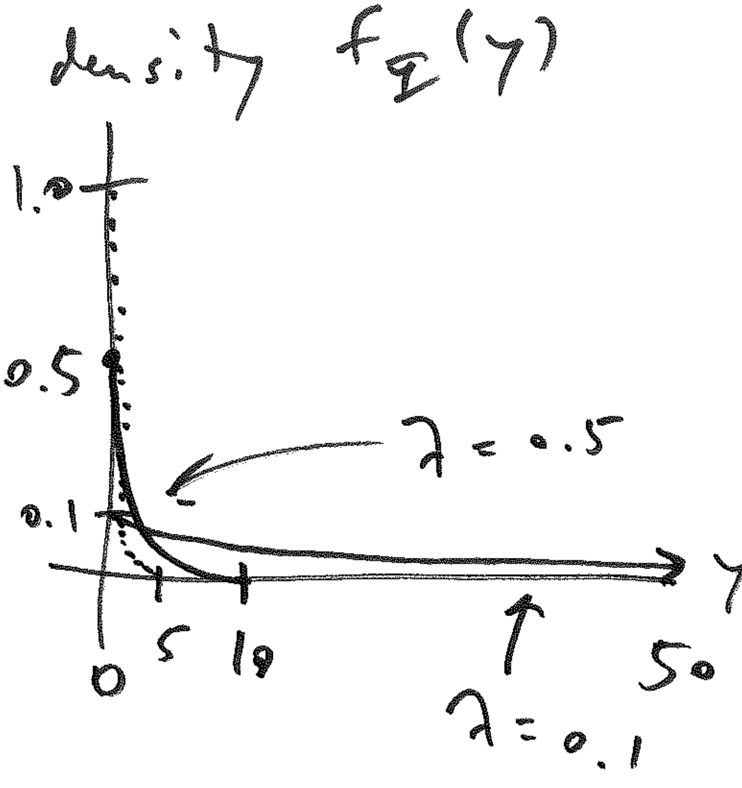
$F_Y^{-1}(p) = -\frac{\log(1-p)}{\lambda}$

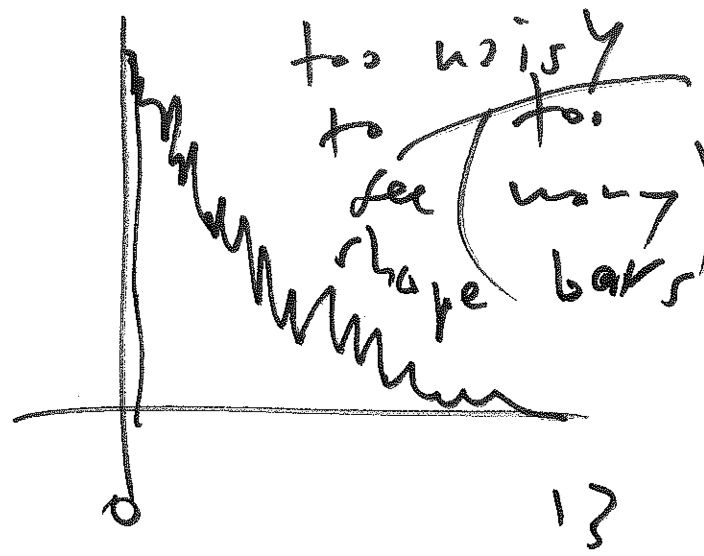












for all λ, Exponential(λ) has a long right-hand tail, but skewness $\downarrow 0$ as $\lambda \downarrow 0$;

mean of Exp.$(\lambda) = \frac{1}{\lambda}$

as $\lambda \uparrow$, center of dist $\downarrow$

as $\lambda \downarrow$, spread $\uparrow$

Cauchy distribution

$$f_Y(y) = \frac{1}{\pi\gamma\left[1 + \left(\frac{y-\mu}{\gamma}\right)^2\right]}$$

μ = center (location), γ = spread (scale)

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let's take ($\mu = 0$, $\gamma = 1$):

$$f_Y(y) = \frac{1}{\pi(1+y^2)}$$

(standard Cauchy)

~~Inverse~~ CDF

$$F_Y(y) = \frac{1}{\pi}\tan^{-1}\left(\frac{y-\mu}{\gamma}\right) + \frac{1}{2}$$

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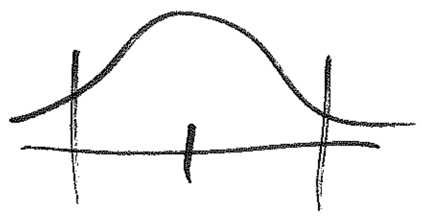
$$F_Y^{-1}(p) = \mu + \gamma \tan\left[\pi\left(p - \tfrac{1}{2}\right)\right]$$

$(\mu = 0, \gamma = 1)$ $\quad F_Y^{-1}(p) = \tan\left[\pi\left(p - \tfrac{1}{2}\right)\right]$

$$F_Y^{-1}(0.25) = \tan\left(-\frac{\pi}{4}\right) = -1$$

$$F_Y^{-1}(0.75) = \tan\left(\frac{\pi}{4}\right) = +1$$

so this distribution has interquartile range 2



0

$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right]$$

$$F_Y(y) = \Phi\left(\frac{y-\mu}{\sigma}\right) = p$$

iff $\Phi^{-1}(p) = \frac{y-\mu}{\sigma} \leftrightarrow y = \mu + \sigma\Phi^{-1}(p)$

$$F_Y^{-1}(p) = \mu + \sigma\, \Phi^{-1}(p)$$

$\boxed{(\mu = 0)}$ $\quad F_Y^{-1}(0.25) = \sigma \underbrace{\Phi^{-1}(0.25)}_{-0.6745}$

so $IQR = 2(0.6745)\,\sigma$

$= 2$ iff $\sigma = \dfrac{1}{0.6745}\,\sigma = 1.4826$

$$f_Y(y) = \frac{1}{\pi(1+y^2)} \qquad \int_{-\infty}^{\infty} dy = 1$$

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but $\displaystyle\int_0^{\infty} \frac{y}{\pi(1+y^2)}\, dy = +\infty$