Disc. sec. 5

DS # 8 p. 141

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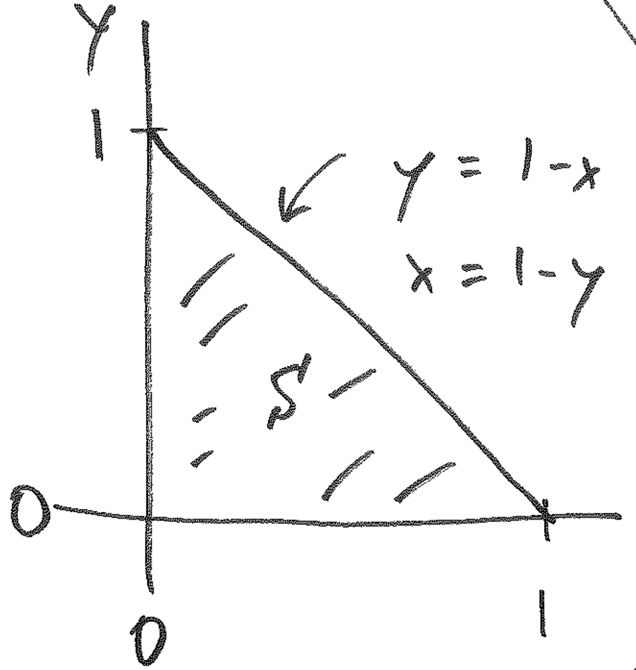
(1)

$$f_{XY}(x,y) = \begin{cases} 24xy & \text{for } x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0 & \text{else} \end{cases}$$



$$\int_0^1 \int_0^{1-x} 24xy \, dy \, dx$$

$$\circledast = 24 \int_0^1 x \left[\int_0^{1-x} y \, dy \right] dx$$

$$\int_0^{1-x} y \, dy = \frac{y^2}{2} \Bigg|_0^{1-x} = \frac{(1-x)^2}{2} - 0$$

$$\circledast = \overset{12}{\cancel{24}} \int_0^1 x \frac{(1-x)^2}{\cancel{2}} dx \quad \ldots \quad = 1$$

$$x(1-2x+x^2)$$

②

$$f_X(x) = \int_0^{1-x} f_{XY}(x,y)\,dy$$

$$= \int_0^{1-x} 24\,xy\,dy$$

$$= \begin{cases} 12\,x\,(1-x)^2 & \text{for } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

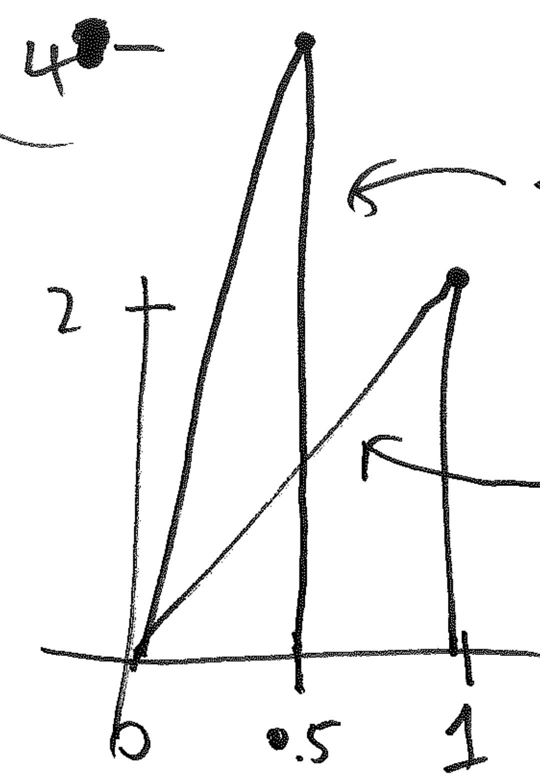
$$f_Y(y) = \int_0^{1-y} 24\,xy\,dx = f_X(y)$$

$$= \begin{cases} 12\,y\,(1-y)^2 & \text{for } 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

← as long as this is > 0

$$= \begin{cases} \dfrac{24xy}{12x(1-x)^2} = \dfrac{2y}{(1-x)^2} & \text{for } x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0 & \text{else} \end{cases}$$



$f_{Y|X}(y|x = \frac{1}{2})$

$f_{Y|X}(y|x = 0)$

$x = \frac{1}{2}$

$x + y \leq 1$

$y \leq \frac{1}{2} \leftarrow \frac{1}{2} + y \leq 1$

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

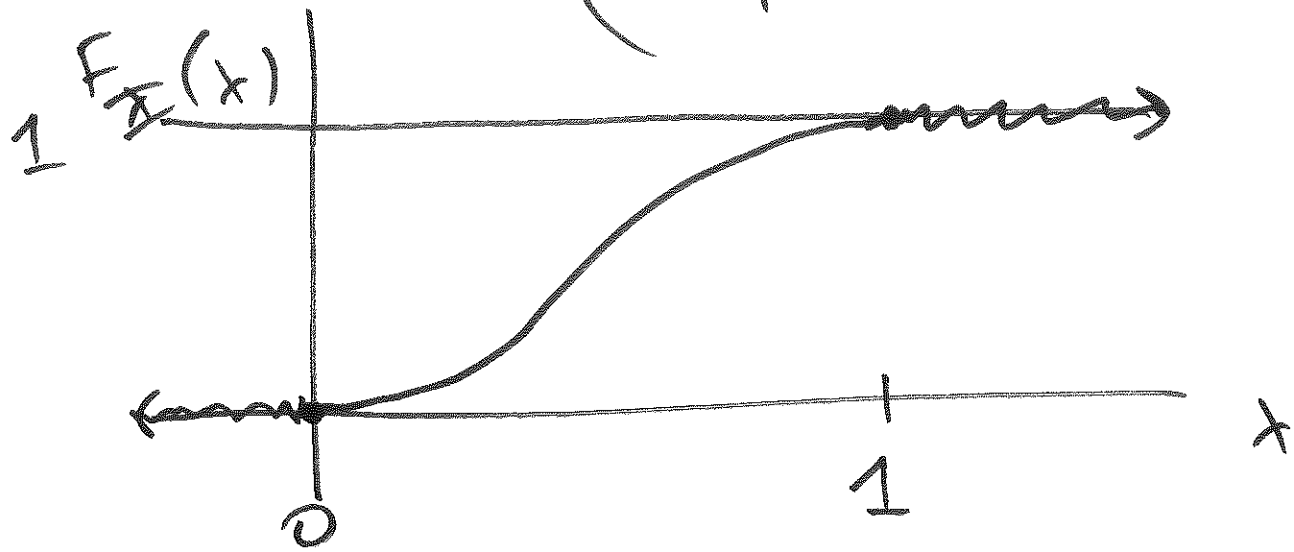
$$= \begin{cases} \dfrac{24xy}{12y(1-y)^2} = \dfrac{2x}{(1-y)^2} & \text{for } x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0 & \text{else} \end{cases}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

$$= \begin{cases} 0 & \text{for } x \leq 0 \\ \int_0^x 12t(1-t)^2\,dt & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

so CDF for x is

$$F_X(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 3x^4 - 8x^3 + 6x^2 & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$



$\cancel{p(N \mid \theta) = \binom{N}{m} \theta^m (1-\theta)^{N-m}}$

$$p(N = n \mid \theta) = \begin{cases} \binom{m}{n} \theta^n (1-\theta)^{m-n} & \text{for } n = 0, \dots, m \\ 0 & \text{else} \end{cases}$$

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$$f_{\Theta}(\theta) = c\ \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$= \text{Beta}(\alpha, \beta) \text{ dist. for } \theta$$