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| 1 Aug 17 |

Disc. sec. 1 | DS p. 15 #8 | $S = \{ A, B, AB, O \}$ | 1

$A_* = \{$ blood reacts with anti-A$)$ ← T-F
$= \{ A, AB \} = (A$ or $AB)$ ← cset

$B_* = \{$ blood reacts with anti-B$)$

$= \{ B, AB \} = (B$ or $AB)$

$A_*^c = \{ B, O \}$ | $B_*^c = \{ A, O \}$

$\{A\} = (A_*$ and $B_*^c)$ | $\{B\} = (B_*$ and $A_*^c)$

$\{AB\} = (A_*$ and $B_*)$ | $\{O\} = (A_*^c$ and $B_*^c)$

DS p. 21 #14 | $P(O) = 0.5$ | $P(A) = 0.34$

$P(B) = 0.12$ | (a) $P(A_*) = P(A$ or $AB)$

②

$P(A_*) = P(A \text{ or } AB) \overset{\text{no overlap}}{=} P(A) + P(AB)$

$P(AB) = 1 - P(A) - P(B) - P(o)$

$= 1 - 0.34 - 0.12 - 0.5$

$= 0.04$

$\therefore P(A_*) = P(A) + P(AB)$

$= 0.34 + 0.04$

$= 0.38$

$P(B_*) = P(B \text{ or } AB)$

$\overset{\text{no overlap}}{=} P(B) + P(AB) = 0.12 + 0.04 = 0.16$

(b)

$P(A_* \text{ and } B_*) = P(AB) = 0.04$

DS p. 35 #1

$P(\text{sum of 2 fair dice is odd}) = \frac{1}{2}$

ELM

1-6

intuition

$S = \{(1,1), (1,2), \ldots, (6,6)\}$

die 1 die 2

sum

die 2

die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

ELM? ✓

$$P(\text{odd}) = \frac{18}{36} = \frac{1}{2}$$

$\boxed{2}$

$$P(\text{even}) = 1 - \frac{1}{2} = \frac{1}{2}$$

$\boxed{3}$ diff (1-2) die 2

die 1	1	2	3	4	5	6
1	0	-1	-2	-3	-4	-5
2	1	0	-1	-2	-3	-4
3	2	1	0	-1	-2	-3
4	3	2	1	0	-1	-2
5	4	3	2	1	0	-1
6	5	4	3	2	1	0

$$P(|\text{die}_1 - \text{die}_2| < 3)$$

$$= 1 - \frac{12}{36} = \frac{2}{3}$$

$\boxed{6}$ $P(\text{all 3 faces same}) = ?$

$S = \{HHH, HHT, \ldots, TTT\}$

$|S| = 2^3 = 8$ $\quad S = \{H, T\}^3$

product space

HHH
HHT
HTH
THH
HTT
THT
TTH
TTT

ELM? ✓ ← fair

$$P(\text{all same}) = \frac{2}{8} = \frac{1}{4}$$

Tay-Sachs case study, in more depth

$P(1 \text{ or more T-S}) = 1 - \left(1 - \frac{1}{4}\right)^5$

$P(0 \text{ T-S}) = \left(\frac{3}{4}\right)^5 = \left(1 - \frac{1}{4}\right)^5$

$Y = \#$ T-S

random variable

y	$P(Y = y)$ possible value
0	$1\left(\frac{1}{4}\right)^0\left(1-\frac{1}{4}\right)^5 \doteq 0.237$
1	$5\left(\frac{1}{4}\right)^1\left(1-\frac{1}{4}\right)^4 = 0.396$
2	$10\left(\frac{1}{4}\right)^2\left(1-\frac{1}{4}\right)^3 = 0.264$
3	$10\left(\frac{1}{4}\right)^3\left(1-\frac{1}{4}\right)^2 = 0.088$
4	$5\left(\frac{1}{4}\right)^4\left(1-\frac{1}{4}\right)^1 = 0.015$
5	$1\left(\frac{1}{4}\right)^5\left(1-\frac{1}{4}\right)^0 \doteq .001$
	1.0

$\doteq$ is approx. equal to

$P(Y = 5) = P\left(\left(\begin{matrix}\text{T-S}\\ \text{on}\\ 1^{st}\end{matrix}\right) \underline{\text{and}} \left(\begin{matrix}\text{T-S}\\ \text{on}\\ 2^{nd}\end{matrix}\right) \underline{\text{and}} \ldots \left(\begin{matrix}\text{T-S}\\ \text{on}\\ 5^{th}\end{matrix}\right)\right)$

I ID

$= P\left(\begin{matrix}\text{T-S}\\ \text{on}\\ 1^{st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{T-S}\\ \text{on}\\ 2^{nd}\end{matrix}\right) \cdots P\left(\begin{matrix}\text{T-S}\\ \text{on } 5^{th}\end{matrix}\right)$

I ID

$= \left(\frac{1}{4}\right)^5$

$S = \{NNNNN, NNNNT, \ldots, TTTTT\}$

T = T-5

N = not T-5

$|S| = 2^5 = 32$

NNNNN	0
NNNNT	
NNNTN	
NNTNN	1
NTNNN	
TNNNN	
NNNTT	
⋮	2
TTNNN	

$\left.\begin{matrix} NNNTT \\ \vdots \\ TTNNN \end{matrix}\right\} = 10$

$P(NNNNT) = \left(\frac{1}{4}\right)^1\left(1-\frac{1}{4}\right)^4$

$P(X = 1)$

$P(T) = \frac{1}{4}$

$P(N) = \frac{3}{4}$

$$
\begin{matrix}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{matrix}
$$

an example of the **Binomial distribution**