This continuity

time: correction,
next confidence

time: intro Is,
Markov chains

Read: 25 pp. 188-201
Fri: Lúpulo Santa Cruz 4:15 pm

Please complete the online course evaluation (+ bump)

(9.51) 1 Please try to upload everything
(10 quizzes, 3 THT) by 1 minute before midnight on Fri 1 Sep.
2 If necessary email PDF to our excellent & patient grader Priyanshu Sharma <psharma@usc.edu>

by 1 minute before midnight Sun 3 Sep.
The confidence interval story

Neyman (~1930)

The single most important frequentist inferential idea

\[ \mathbf{X}, \ldots, \mathbf{X}_n \sim \text{some dist.} \]

\[ E(\mathbf{X}_i) = \mu \]

\[ V(\mathbf{X}_i) = \sigma^2 \]

For \( n \) large,

\[ \frac{1}{n} \sum_{i=1}^{n} \mathbf{X}_i = \overline{\mathbf{X}}_n \]

\( \overline{\mathbf{X}}_n \xrightarrow{\text{CLT}} N(\mu, \sigma^2/n) \]

\[ \frac{\overline{\mathbf{X}}_n - \mu}{\sigma/\sqrt{n}} \sim \text{dist. (PDF)} \]

\( \hat{\theta} \) is an estimator of \( \theta \) of interest

\[ \hat{\theta} \xrightarrow{\text{large}} \theta \]

\( \overline{\mathbf{X}}_n \xrightarrow{\text{PDF dist.}} \theta \)
yet what people need: they need
something like $P(A < \theta < B) = \beta$.
Neyman's confidence trick

\[ .95 = P_\theta \left( \hat{\theta}_n - 2\hat{SE} < \theta < \hat{\theta}_n + 2\hat{SE} \right) \]

rewritten \[ \hat{\theta}_n - 2\hat{SE} < \theta \]

rewritten \[ \hat{\theta}_n > \theta - 2\hat{SE} \]

\[ \theta < \hat{\theta}_n + 2\hat{SE} \]

\[ .95 = P_{\theta_1} - 2{\theta_2}^{\hat{SE}} < \theta < \hat{\theta}_n + 2{\theta_2}^{\hat{SE}} \]

\[ A \quad B \]

r.v.s

Therefore let's agree to call \[ \hat{\theta}_n \pm 2\hat{SE} \] an approximate 95% confidence interval for \( \theta \) (CI).
Approximate 99.9% confidence interval for $\theta$ is:

$$\hat{\theta}_n \pm 3.3 \frac{s}{n}$$

Approximate 95% confidence interval is:

$$\hat{\theta}_n \pm t_{1 - \frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

For simplicity, let $s = \frac{s}{\sqrt{n}}$.

The probability $P_F(1 < \theta < 2) = 0.95$ is undefined.

$P_F(\text{hit}) = 0.95$

$\theta$: includes $\theta$.

Significant hits:

- hit
- miss
- hit
- hit
Every estimator $\hat{\theta}_n$ is a r.v. so it has a variance $\text{Var}(\hat{\theta}_n)$; $\text{SE} = \sqrt{\text{Var}(\hat{\theta}_n)}$. Unfortunately, $\sqrt{\text{Var}(\hat{\theta}_n)}$ often involves $0$ when this occurs, define $\text{SE}$ to be $\sqrt{\text{Var}(\hat{\theta}_n)}$. stick in $\hat{\theta}_n$ whenever $\theta$ appears.

$$\Leftrightarrow \text{99.9}\% \text{ CE for } (\hat{\theta}_c - \hat{\theta}_t)$$

where $\theta$ is not in CE, difference is (statistically) significant.
PDF of $\bar{X}_n$

$\mu_X$

$n$ large

$SD(\bar{X}_n) = \frac{\sigma_X}{\sqrt{n}}$

$SE(\bar{X}_n)$

$\sigma_X$ unknown, so $SE(\bar{X}_n) = \frac{s_X}{\sqrt{n}}$

is undesirable; natural fix: estimate sample mean

Note $\bar{X}_n$ is a good estimate of $\mu_X$

$s_X = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2}$ is a good sample est. of $\sigma_X \leq 1.1s$

so let's work with $SE(\bar{X}_n) = \frac{s_X}{\sqrt{n}}$
Sample

The observed
hypotensive
people

\[
\text{diff} (B - A) = D
\]

\[
\begin{array}{c}
D_1 = 9 \\
D_2 = 13 \\
D_3 = 12 \\
D_4 = 18 \\
D_5 = 14 \\
D_6 = 21 \\
D_7 = 16 \\
D_8 = 12 \\
D_9 = 11 \\
D_{10} = 18 \\
D_{11} = 15 \\
D_{12} = 16
\end{array}
\]

\[
\bar{D} = 16.6 \text{ mm Hg}
\]

\[
\sigma = ?
\]

\[
\sigma_D = 10.1 \text{ mm Hg}
\]

\[
\text{mean } \bar{D} = 16.6 \text{ mm Hg}
\]

\[
\text{ex. (19.3)}
\]

\[
\text{(estimate of } \Delta) = \hat{\Delta} = \bar{D}_n = 16.6
\]

\[
\text{take for } \Delta \text{ as est. of } \Delta
\]

\[
\text{95% CI for } \Delta
\]

\[
16.6 \pm 2 \left( \frac{10.1}{\sqrt{12}} \right) = 16.6 \pm 2(2.9) = 16.6 \pm 6
\]
$95\% \text{ CI for } \Delta$

\[
\begin{array}{c}
13 \\
19 \\
25
\end{array}
\]

Proving that

\[ P(13 < \Delta < 25) = 0.95 \]

is undefined.

Imagine lots of CI, each based on $h = 12$ PID draws from $p_1$.

$P_r(\text{hit}) = 0.95$