AMS131
30 Aug 17

this time: continuity correction,
next time: confidence intervals,
Markov chains

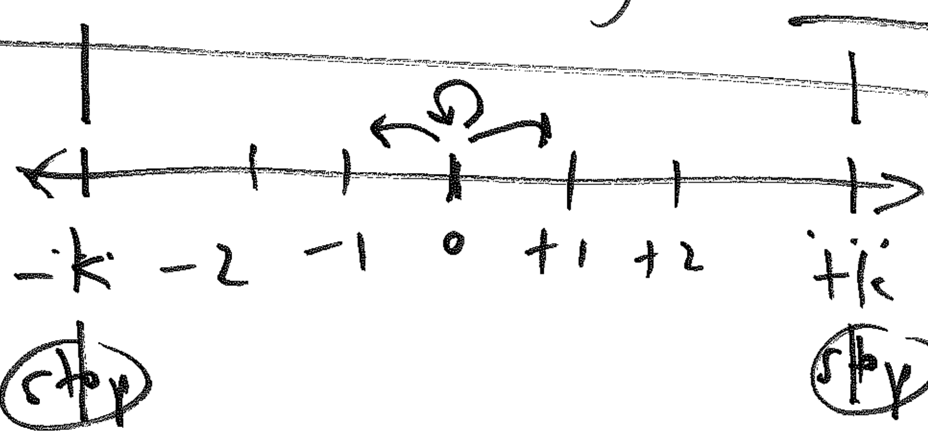
Read: DS pp. 188–201 & sec. 8.5

Fri 1 Sep: Lúpulo Santa Cruz 4–6 pm

①

please complete the online course evaluation (+ bump)

(9.51) ① Please try to upload <u>everything</u> (10 quizzes, 3 THT) by 1 minute before midnight on Fri <u>1</u> Sep. ② If <u>necessary</u> email PDF to our excellent & patient' grader Priyanshu Sharma <psharma8@ucsc.edu> by 1 minute before midnight <u>Sun</u> 3 Sep.



The confidence interval story — single most important frequentist inferential idea

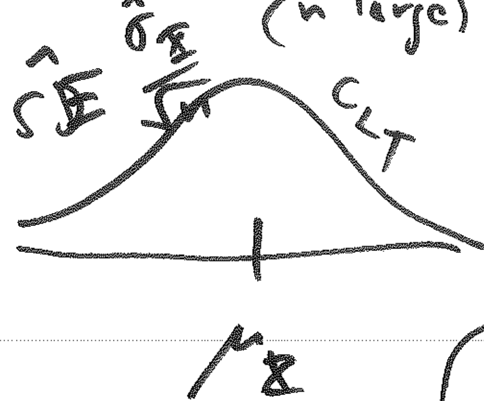
Neyman (~ 1930)

$X_1, \dots, X_n \overset{IID}{\sim}$ some dist. $(\sigma_X^2 < \infty)$

$E(X_i) = \mu_X$, $V(X_i) = \sigma_X^2$

for n large,

$$\boxed{\frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}_n}$$



dist. (PDF) of $\bar{X}_n$

$$\boxed{\bar{X}_n \xrightarrow{P} \mu_X}$$

$X_1, \dots, X_n \overset{IID}{\sim}$ some dist, $(\sigma_X^2 < \infty)$

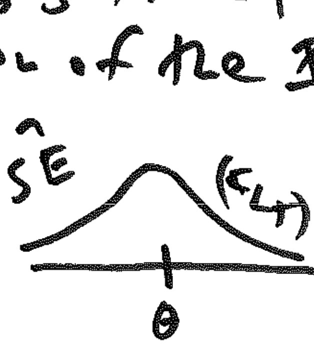
(summary of dist)

unknown parameter θ of interest

suppose you can find an estimator $\hat{\theta}_n$ of

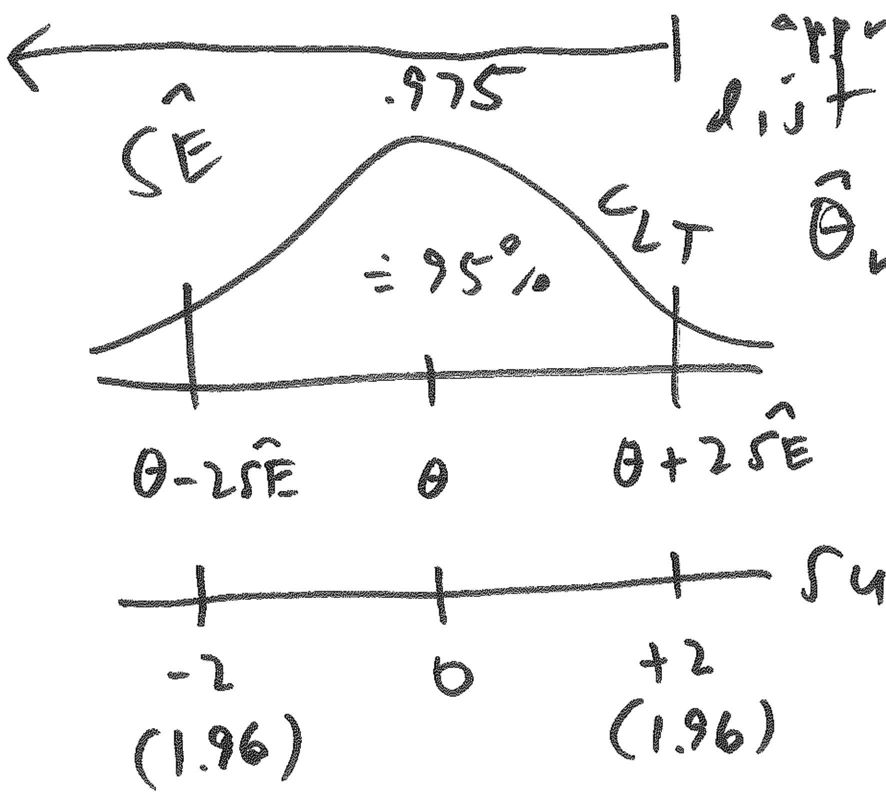
(a function of the X_i)

θ such that



(PDF) dist. of $\hat{\theta}_n$

(n large)



approx. dist. f $\hat{\theta}_n$

Neyman was a rabid frequentist: for him θ is a fixed unknown constant and $\hat{\theta}_n$ is a r.v.

fixed   fixed   r.v.   fixed   fixed

$$P_F\left(\theta - 2\hat{SE} < \hat{\theta}_n < \theta + 2\hat{SE} \right) \doteq 95\%$$

frequentist

This is not yet what people need; they need something like $P_B\left(A < \theta < B \,\middle|\, \text{data} \right) \doteq .95$

if A, B are fixed

$$P_F(A < \theta < B) = \text{undefined}$$

fixed known   fixed unknown   fixed known

$\boxed{\text{Neyman's confidence trick}}$

$$.95 = P_F\left(\underline{\theta - 2\hat{SE}} < \hat{\theta}_n < \underline{\theta + 2\hat{SE}} \right)$$

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rewrite $\hat{\theta}_n < \theta + 2\hat{SE} \leftrightarrow$

$$\hat{\theta}_n - 2\hat{SE} < \theta$$

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rewrite $\hat{\theta}_n > \theta - 2\hat{SE} \leftrightarrow$

$$\theta < \hat{\theta}_n + 2\hat{SE}$$

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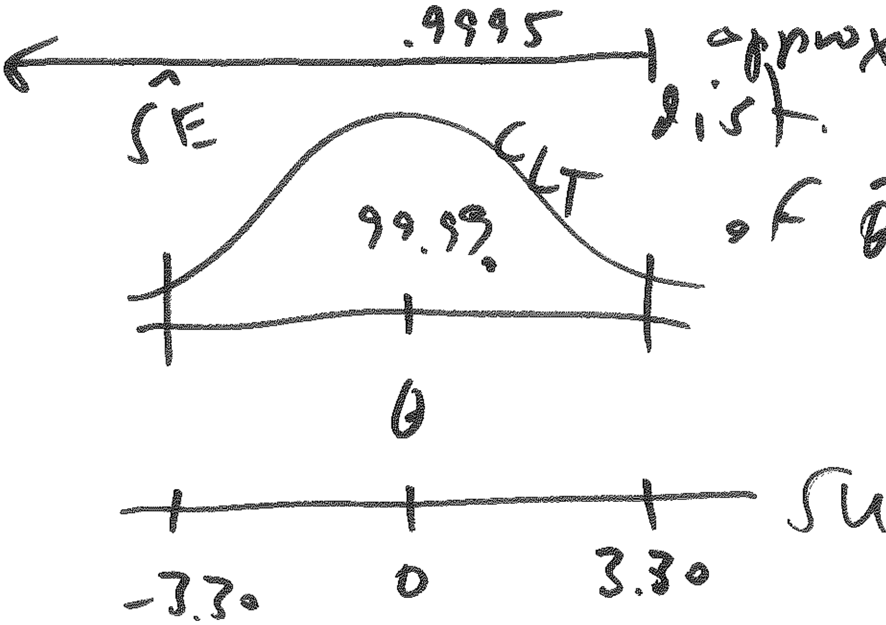
r.v. fixed r.v.

$$.95 = P_F\left(\underbrace{\hat{\theta}_n - 2\hat{SE}}_{A} < \theta < \underbrace{\hat{\theta}_n + 2\hat{SE}}_{B} \right)$$

A, B r.v.s

Therefore let's agree to call

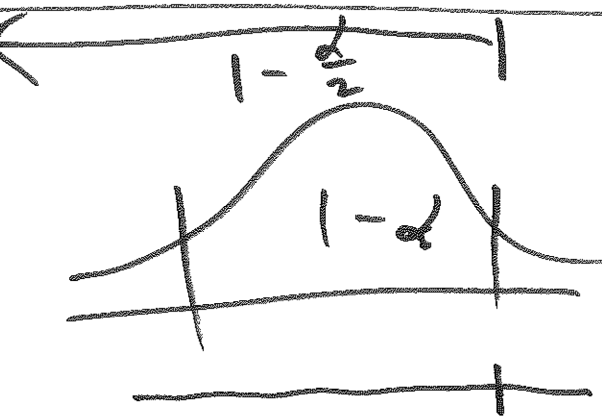
$\boxed{\hat{\theta}_n \pm 2\hat{SE}}$ an $\boxed{\text{~~approximate~~ 95\% confidence interval for } \theta}$ (CI)



approx. dist. of $\hat{\theta}_n$

An approx. 99.9% CI for θ is

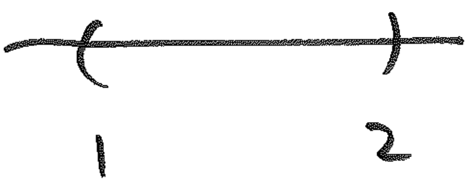
$$\hat{\theta}_n \pm 3.3\,\hat{SE}$$



An approx. $100(1-\alpha)\%$ CI is

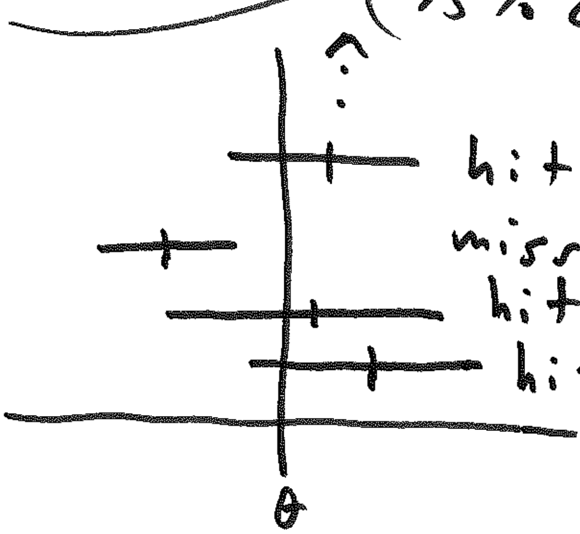
$$\hat{\theta}_n \pm \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\cdot \hat{SE}$$

95% CI for θ



$P_F(1 < \theta < 2) \doteq \cancel{.95}$ is undefined

(95% CIs)



$P_F(\text{hit}) \doteq .95$

hit = includes θ

Every estimator $\hat{\theta}_n$ is a r.v.

so it has a variance $V(\hat{\theta}_n)$;

$\boxed{SE \triangleq \sqrt{V(\hat{\theta}_n)}}$ unfortunately

$\sqrt{V(\hat{\theta}_n)}$ often involves θ

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when this occurs, define $\hat{SE}$ to

be $\sqrt{\hat{V}(\theta_n)}$ stick in $\hat{\theta}_n$ wherever θ appears

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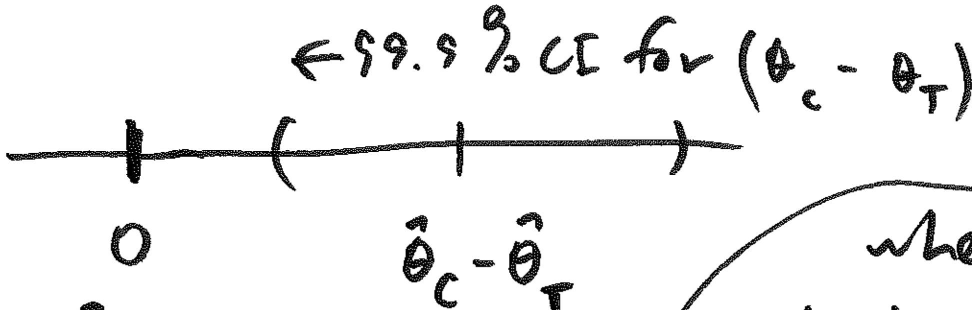
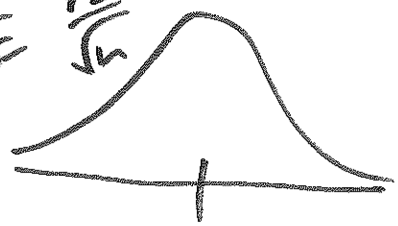
(10.43)



devil's advocate

when 0 is not in CI, diff. is statistically significant (statsig)

$\hat{SE}$ $\frac{\cancel{\sigma_X}\; s_X}{\sqrt{n}}$



μ_X

PDF of $\bar{X}_n$

n large

$$\boxed{SD(\bar{X}_n) = \frac{\sigma_X}{\sqrt{n}}}$$

$$= SE(\bar{X}_n)$$

σ_X unknown, so $SE(\bar{X}_n) = \frac{\sigma_X}{\sqrt{n}}$ is unusable; $\boxed{\text{natural fix:}}$ in the same way that $\bar{X}_n$ (sample mean) is a good estimate of μ_X (pop. mean),

$$s_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

is a good est. of σ_X ← pop. SD

sample SD

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so let's work with $\hat{SE}(\bar{X}_n) = \frac{\cancel{\sigma_X}\; s_X}{\sqrt{n}}$

8

pop: all UK adult hypertensives in 1975

diff (B − A)

$N = ?$ (big)

mean $\Delta = ?$

SD $\sigma_D = ?$

sample: the observed hyp. people

diff $(B-A) = D$

actual

like IID

D_1 $d_1 = +9$

⋮

D_n $d_{12} = +33$

$n = 12$

mean $\bar{D}_n = 18.6$ mmHg

SD $s_D \doteq 10.1$ mmHg

hyp. IID

θ

$n = 12$

mean $\bar{D}_n = ?$ ex. (19.3)

repeated sampling

18.6
19.3
⋮

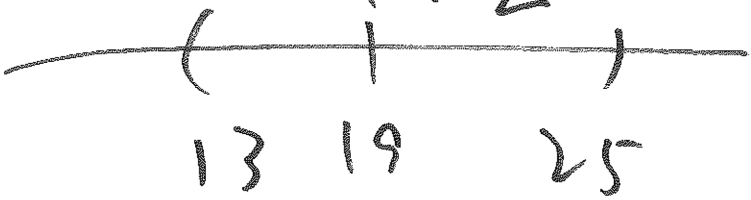
M

$$\begin{pmatrix}\text{estimate} \\ \text{of } \Delta\end{pmatrix} = \hat{\Delta} = \bar{D}_n = 18.6$$

$$\begin{pmatrix}\text{give or} \\ \text{take for} \\ \hat{\Delta} \text{ as est.} \\ \text{of } \Delta\end{pmatrix} = \widehat{SE}(\hat{\Delta}) = \frac{s_D}{\sqrt{12}} = \frac{10.1 \text{ mmHg}}{\sqrt{12}} \doteq 2.9$$

$$\begin{pmatrix}95\% \text{ CI} \\ \text{for } \Delta\end{pmatrix} = 18.6 \text{ mmHg} \pm 2\,(2.9 \text{ mmHg}) \simeq 19 \pm 6$$

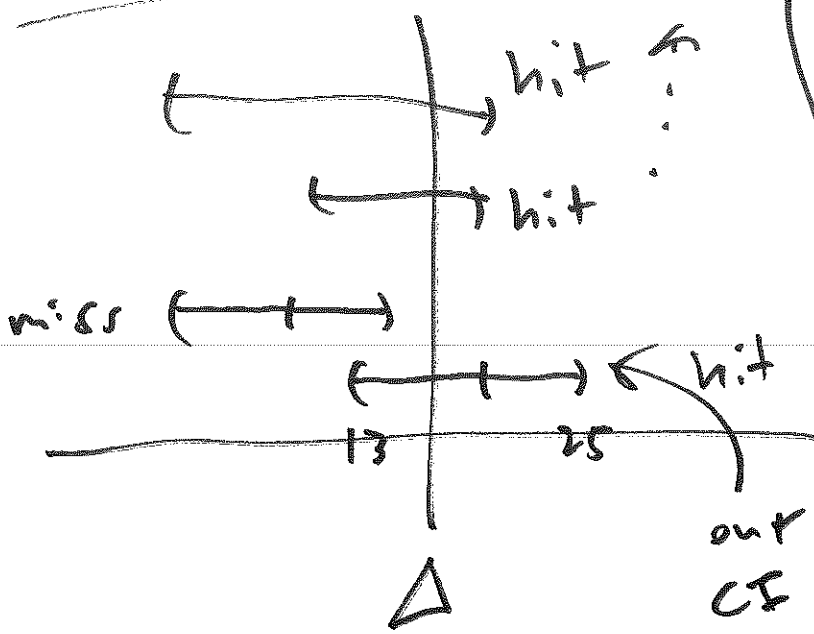
95% CI for Δ



does this mean that

$$P_{F}(13 < \Delta < 25) \doteq \cancel{0.95} \; ?$$

$$\doteq \text{undefined}$$



$$P_F(\text{hit}) \doteq 0.95$$

imagine lots of CI, each based on $n = 12$ IID draws from p.p.

(10.58)