this time: joint, marginal & conditional distributions

read DS. ch 3, 4

AMS 131
11 Aug 17

next time: expectation

let's compute ① the quantile function (inverse CDF) of the Exponential distribution for $\lambda > 0$

$(Y|\lambda) \sim \text{Exponential}(\lambda)$

$\leftrightarrow f_Y(y) = \lambda e^{-\lambda y} I(y > 0)$ and

$$F_Y(y) = (1 - e^{-\lambda y}) I(y > 0)$$

all we have to do is set $1 - e^{-\lambda y_p} = p$ and solve for y_p: $1 - p = e^{-\lambda y_p}$, so

(ln)
$\log(1-p) = -\lambda y_p$ and

$$\boxed{y_p = F_Y^{-1}(p) = -\frac{1}{\lambda}\log(1-p) \quad \text{for } 0 < p < 1}$$

Ex. $\lambda = 0.25$; median of the Exponential dist. with $\lambda = 0.25$ is $y_{0.5} = -\frac{\log(1-0.5)}{0.25} \doteq 2.77$

turns out that mean is $\frac{1}{\lambda} = 4$

$$F_Y(y) = P(Y \le y)$$

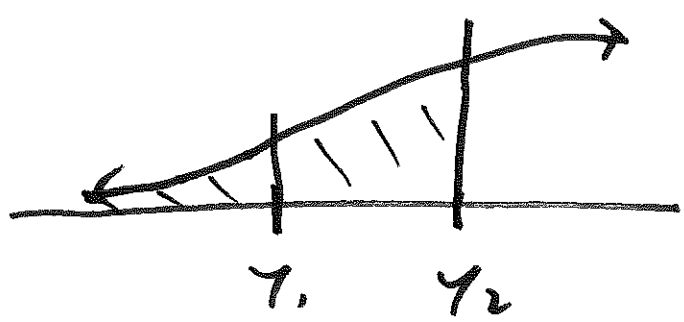
$$\underbrace{P(Y \le y)}_{F_Y(y)} + P(Y > y) = 1$$

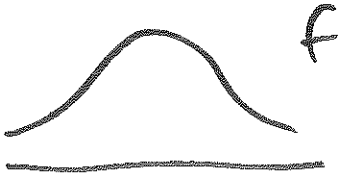
$$\therefore \; P(Y > y) = 1 - F_Y(y)$$

$$F_Y(y_1) = P(Y \le y_1)$$

$$F_Y(y_2) = P(Y \le y_2)$$

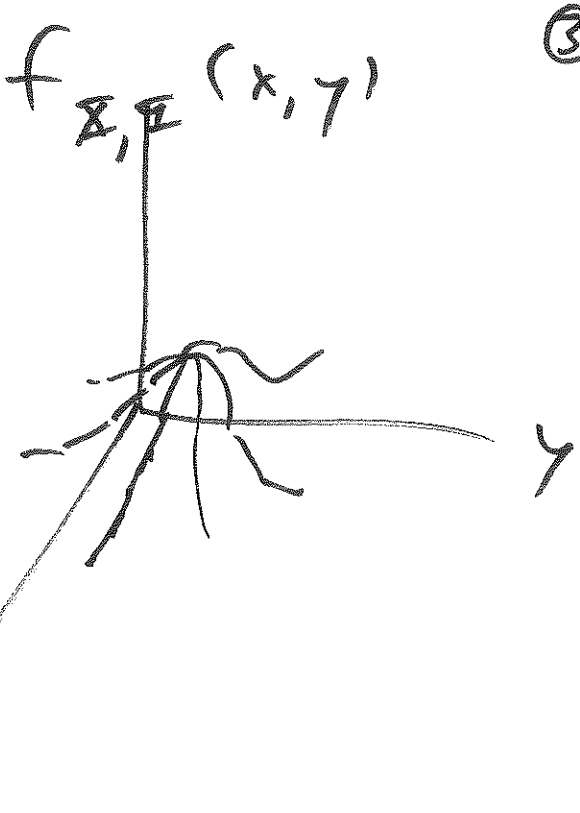
$y_1 < y_2$

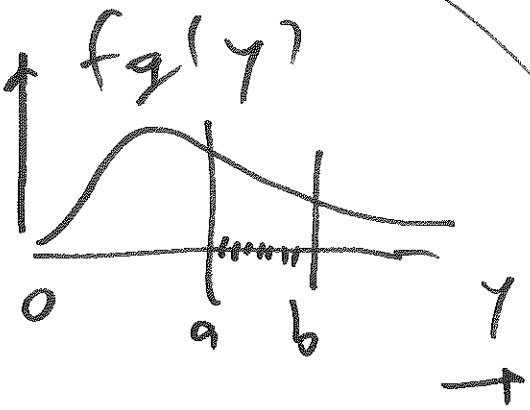




$f_X(x)$

$f_Y(y)$



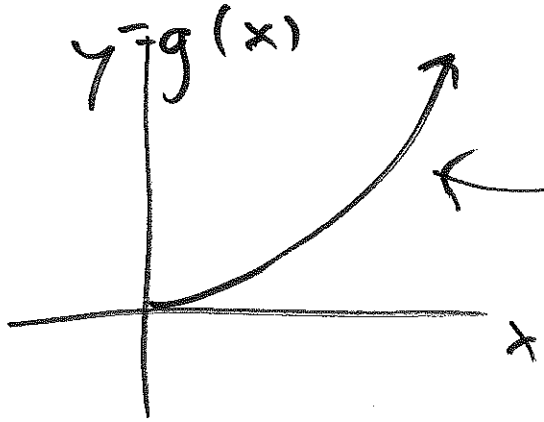


Q is 1-dimensional

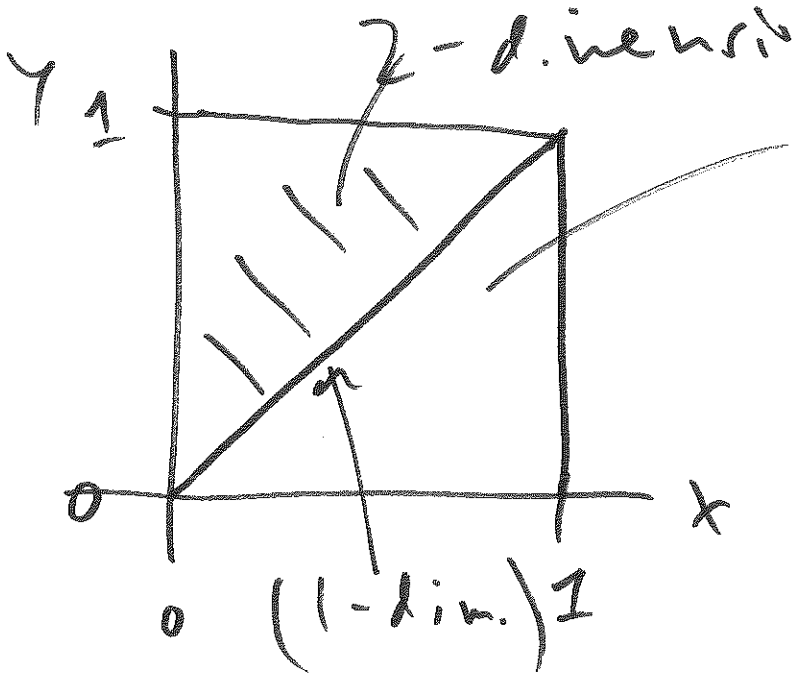
Y continuous on $(0,\infty)$

$P(Y = 1) = 0$

unsingleton is 0-dimensional



1-dimensional (prob. 0)



support of Uniform dist. on the unit square

$$P(Y = X) = 0$$

$$P(Y > X) > 0 = \frac{1}{2}$$

population

all deer living at Wegscott Ranch

1 = y
0 = n

disease?

0
0
⋮
1
⋮
0

N = 800

mean θ = ?

(unknown)

at random

(now for each deer)

sample

the observed deer

disease?

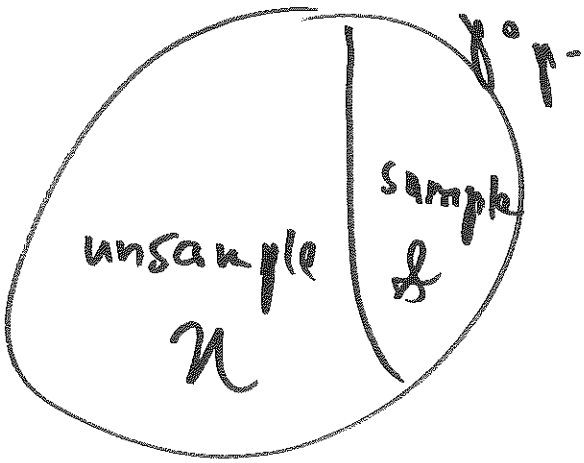
1s
&
0s

n = 142

mean $\bar{y} = \frac{3}{142} \doteq 2\%$

unknown(s) → data
(prob.)

data → unknown(s)
(inference)



goal: want $\mathcal{S}$ & $\mathcal{U}$ as **similar** as possible in all **relevant** way

method: choose $\mathcal{S}$ at random

pop. **prob.** easier
(whole) deduction sample (part)
(general) $\longrightarrow$ (particular)
at random

induction = statistical inference

statistics (harder)