AMS 131 extra notes 1

Definition An *experiment* $\mathcal{E}$ is a data-generating process in which all possible outcomes can be listed before $\mathcal{E}$ is performed.

Definition An *event* E is a set of possible outcomes of an experiment $\mathcal{E}$.

Example Tay-Sachs (T-S) disease

$\mathcal{E}$ = (the process by which the husband & wife end up with 5 children, each a T-S baby or not)

the E of interest is $E = \{$at least 1 T-S baby$\}$

Definition The sample space S is the set of all possible outcomes (elements) of an experiment $\mathcal{E}$.

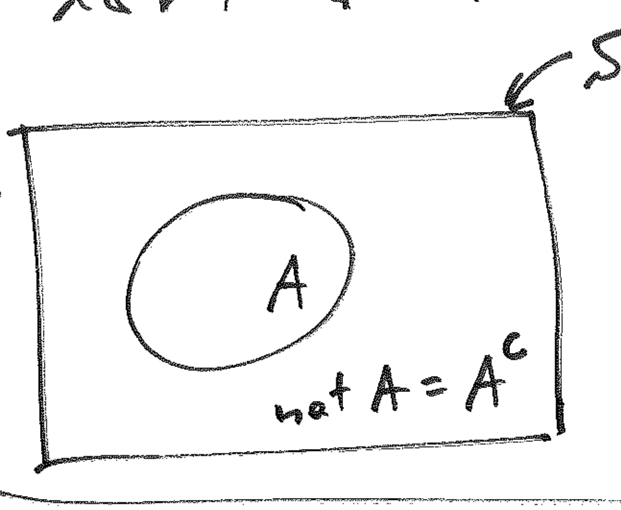
Example: T-S

Let T = (T-S baby) and N = (not T-S baby)

NNNNN	Here $S = \{NNNNN, \ldots, TTTTT\}$
TNNNN	
NTNNN	Since there are 2 possibilities for each baby (T, N) and 5 babies, the number of elements in S is $2^5 = 32$.
NNTNN	
NNNTN	
NNNNT	
TTNNN E	
TNTNN	S is an example of a **product space**:
⋮	$\underbrace{\{T,N\} \times \{T,N\} \times \cdots \times \{T,N\}}_{5} = \{T,N\}^5$.
TTTTT	

Here $E = \{TNNNN, \ldots, TTTTT\}$.

Notation Let's use s to stand for the individual outcomes (elements) of $\mathcal{S}$. The theory of probability we'll look at in this class was developed by Kolmogorov (1933) in an attempt to rigorize the hypothetical process of repeatedly throwing a dart at a Venn diagram (rectangle)



The rules of this dart-throwing were simple: ① the dart must land somewhere inside (or on the boundary of) the rectangle $\mathcal{S}$, which symbolically stands for the sample space, and ② all the points where the dart might land in S are "equally likely" (as yet, an undefined primitive concept).

Definition The complement A^c of a set A contained in S is the set that contains all elements of S not in A

(You can see from the Venn diagram on p. ③ that the dart has to fall either in A or in A^c, which we could also call $\boxed{\text{not } A}$.) **Notation**

$s \in S$ means that $\left\{ \begin{matrix} \text{outcome} \\ \text{element} \end{matrix} \right\}$ s belongs to (is an element of) S

Definition | A set A is <u>contained in</u> another set B (A is a <u>subset</u> of B) (written $A \subset B$) if every element of A is also in B; can also say that B <u>contains</u> A ($B \supset A$).

Evidently, if A and B are events,

$A \subset B \overset{\text{(iff)}}{\longleftrightarrow}$ (if and only if) if A occurs then so does B

Consequences (theorem) | If A, B, C are events

then (a) $A \subset B$ and $B \subset A \leftrightarrow A = B$

and (b) $A \subset B$ and $B \subset C \to A \subset C$.

Definition | The <u>cardinality</u> of a set A (written $|A|$) is the number of distinct elements in A.

Example (Tay-Sachs) | $|\mathcal{S}| = 32$ (see p. 3)

Definition The set of all subsets of a given set $\mathcal{S}$ is called the <u>power set</u> of $\mathcal{S}$, denoted by $2^{\mathcal{S}}$; this notation was chosen because, if $|\mathcal{S}| = n$, then $|2^{\mathcal{S}}| = 2^n$ (in other words, if $\mathcal{S}$ has n distinct elements then there are 2^n distinct subsets of $\mathcal{S}$.

Definition It's convenient to have a symbol for the set that has no elements in it: $\emptyset$, the <u>empty set</u>.

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